A Quick Tool to forecast VaR using Implied and Realized Volatilities

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Abstract

We propose here a naive model to forecast *ex-ante* Value-at-Risk (VaR) using a shrinkage estimator between realized volatility estimated on past return time series, and implied volatility extracted from option pricing data. Implied volatility is often indicated as the operators expectation about future risk, while the historical volatility straightforwardly represents the realized risk prior to the estimation point, which by definition is backward looking. In a nutshell, our prediction strategy for VaR uses information both on the expected future risk and on the past estimated risk.

We examine our model, called Shrinked Volatility VaR, both in the univariate and in the multivariate cases, empirically comparing its forecasting power with that of two benchmark VaR estimation models based on the Historical Filtered Bootstrap and on the RiskMetrics approaches.

The performance of all VaR models analyzed is evaluated using both statistical accuracy tests and efficiency evaluation tests, according to the Basel II and ESMA regulatory frameworks, on several major markets around the world over an out-of-sample period that covers different financial crises.

Our results confirm the efficacy of the implied volatility indexes as inputs for a VaR model, but combined together with realized volatilities. Furthermore, due to its ease of implementation, our prediction strategy to forecast VaR could be used as a tool for portfolio managers to quickly monitor investment decisions before employing more sophisticated risk management systems.

Keywords: Value-at-Risk Forecast, Backtest, Shrinkage, Empirical Finance, Market Risk, ESMA, UCITS.

1 Introduction

Financial markets are often affected by recessions and crises. In the last decades we recall the black Monday October 19, 1987, when the Dow Jones fell more than 20% and many quantitative portfolio insurances (OBPI and CPPI) collapsed; or the Black Wednesday September 16, 1992, when the British government was forced to withdraw the pound sterling from the European Exchange Rate Mechanism (ERM). Later in 1997 the UK Treasury estimated the cost of the Black Wednesday at £3.4 billion. At the end of the '90s the Russian government and the Russian Central Bank devalued the ruble and announced the default (Russian crisis 1998). In the last decade three great crises have afflicted the markets: the dot-com bubble (2001-2002), where the Nasdaq index fell more than 70%; the subprime financial crisis with the defaults of large investment banks (2008); and the Eurozone Government Bond crises (2011). A way to be well prepared to manage periods of financial turbulence is groped to predict market risk, estimated by some risk measures. One may use volatility, Value-at-Risk (VaR), Conditional VaR, downside volatility, or others. However, all these indicators should be monitored in order to have an idea of the markets conditions. In financial firms, as banks and asset management companies, the VaR risk measure is commonly used (Jorion, 2007). For instance, the banks must periodically report to their own vigilance authority a VaR estimate of the entire business, along with an accurate backtesting procedure that validates the VaR model used for the estimate.

Many models have been developed to foresee market risk (see, e.g., Abad et al, 2014; Boucher et al, 2014; Louzis et al, 2014, and references therein), taking into account the following stylized facts that characterize the returns time series: volatility clustering, fat tails, and mild skewness (Cont, 2001). Furthermore, VaR models to be accurate should satisfy two conditions: statistical significance when comparing the observed frequency of VaR violations w.r.t. the expected one, and independence of violations (Campbell, 2005).

In this paper, we propose a naive model to forecast *ex-ante* VaR using a shrinkage estimator (Ledoit and Wolf, 2004) between realized volatility estimated on daily return time series, and implied volatility extracted from option pricing data. Indeed, several studies highlight that models based on implied volatility produce competitive VaR forecasts (see, for instance, Giot, 2005; Kuester et al, 2006). Implied volatility is often indicated as the operators expectation about future risk, while the historical-based volatility simply represents the realized risk up to the estimation time, thus employing a backward looking approach.

The purpose of this work is to compare our model, called Shrinked Volatility VaR (ShVolVaR), with several prediction strategies both in the univariate and in the multivariate cases. More in detail, we firstly discuss and analyze three simple models to forecast the one-day-ahead VaR, using implied volatility, realized volatility, and a shrinkage of them. Then we empirically compare their forecasting power with two benchmark VaR models based on Historical Filtered Bootstrap (Barone-Adesi et al, 1999; Bollerslev, 1986; Brandolini et al, 2001; Brandolini and Colucci, 2012; Marsala et al, 2004; Vošvrda and Žikeš, 2004; Zenti and Pallotta, 2000) and on RiskMetrics (Morgan, 1996) approaches over a relatively long time period (at least fourteen years) that depends on the availability of implied volatility values. For these five models, we evaluate the statistical accuracy of one-day-ahead VaR estimates by means of the unconditional coverage test (Kupiec, 1995), which analyzes the statistical significance of the observed frequency of violations w.r.t. the expected one, the independence test (Christoffersen, 1998) which gauges the independence of violations, namely the absence of violation clustering, and the conditional coverage test which combines these two desirable properties (Christoffersen and Pelletier, 2004). In addition to performing tests on accuracy, we check the practical compliance of the VaR models with respect to specific regulatory rules. More precisely, for backtesting aims the European Regulator, i.e., the Committee of European Securities Regulators CESR (now the European Securities and Markets Authority, ESMA), will accept no more than seven violations of $VaR_{1\%}$ (related to a one-day time horizon) on 250-day rolling time windows (CESR, 2010). Furthermore, the one-day ahead VaR should satisfy the coverage condition, while no tests are required by ESMA regarding the independence property of VaR violations. From the viewpoint of the Regulator, a model that overestimates VaR (i.e., it is conservative) is accepted, even though the backtesting shows a high percentage of zero violations, but from the investor viewpoint this means the mismanagement of capital. Conversely, an underestimation of VaR (i.e., the model is aggressive) is convenient for the investor, but it is not accepted from the Vigilance. Therefore, in our backtesting we highlight the right tradeoff between these two different points of view, controlling both the lack and the excess of violations. In other words, the features that a VaR model should satisfy are to minimize on period of 250 days the frequency of absence of violations (the investor viewpoint) and to minimize the frequency that more than seven violations occur (the Regulator viewpoint).

Our results confirm the efficacy of the implied volatility indexes as inputs for a VaR model, but together with realized volatilities. Indeed, implied and realized volatilities, taken individually, are not able to predict VaR violations, and, furthermore, they often fail the accuracy tests. On the other hand, the model based on their shrinkage significantly increases its predictive power of VaR, and also shows that the null hypotheses of independence of VaR violations and the null hypotheses of conditional coverage are usually not rejected.

The contribution of our study can therefore be summarized as follows:

1. we present a simple prediction strategy to model VaR with performance comparable to that of sophisticated simulation models;

2. we provide a tool for portfolio managers that is easy to implement, for example on a common spreadsheet, in order to quickly monitor investment decisions before

passing the tests of more sophisticated risk management systems;

3. we empirically observe that the use of the shrinkage estimator between realized and implied volatilities implicitly tends to satisfy the well-known stylized facts that characterize the returns time series, and works well both in the univariate and in the multivariate contexts;

4. the performance of all VaR models is treated both using statistical accuracy tests and efficiency evaluation tests according to the Basel II and ESMA regulatory frameworks;

5. we analyze the one-day-ahead VaR forecasts performance on several major markets around the world (S&P500, Eurostoxx 50, DAX, FTSE 100 and TOPIX) over an out-of-sample period that covers different financial crises, the Russian crises (1998), the dot-com bubble (2001), the Emerging Markets flash crash (2004), the subprime crises (2008), and the Eurozone Government Bond Crises (2011).

The rest of this paper is structured as follows. Section 2 describes the five models analyzed, and provides the description of the methodologies used to test Unconditional Coverage, Independence and Conditional Coverage, along with the backtesting procedure of Regulator. In Section 3 we illustrate the data sets considered, and discuss the main results of the empirical analysis. Finally, some concluding remarks are drawn in Section 4.

2 Models and Tests

Before introducing the models analyzed in this study to forecast the one-day-ahead Value-at-Risk (VaR), it is useful to specify its mathematical definition. VaR is defined as the maximum loss at a specified confidence level and it is one of the most important risk management tool in the financial industry (Morgan, 1996).

Let us introduce some notations and assumptions. Since we study the VaR performance of the proposed models both in the univariate and the multivariate framework, we use linear returns, so if $p_{t,k}$ is the price of asset k at time t, then $r_{t,k} = \dfrac{p_{t,k} - p_{t-1,k}}{p_{t-1,k}}$ represents its return at time t. Even though for econometric models the returns are usually defined as log-returns, namely $r^{\ln}_{t,k} = \ln p_{t,k} - \ln p_{t-1,k}$, in case of assets portfolios the linear returns are preferred to the logarithmic ones, due to their mathematical tractability. In addition, for small values of $r_{t,k}$, as in this context, it is straightforward to demonstrate that $r_{t,k} \simeq r^{\ln}_{t,k}$.

We denote by $x = (x_1, x_2, \ldots, x_n)^T$ the vector of the assets weights in a portfolio. Thus assuming that n assets are available in an investment universe, the portfolio return at time t $R_t(x) = \sum_{k=1}^{n} x_k r_{t,k}$. Furthermore, the set of feasible portfolios considered in this study satisfy the budget constraint ($\sum_{k=1}^{n} x_k = 1$) and the no short-selling condition ($x_k \geq 0$ for all $k = 1, \ldots, n$).

That being said, VaR_ε is defined as the minimum level of loss at a given confidence level related to a predefined time horizon. Usually, the confidence level are 95% and 99%, that is in general equal to $(1-\varepsilon)100\%$. Hence, $VaR_\varepsilon(x)$ is the value such that the possible portfolio loss $L(x) = -R(x)$ exceeds $VaR_\varepsilon(x)$ with a probability of $\varepsilon 100\%$ (Acerbi and Tasche, 2002). In other words, $VaR_\varepsilon(x)$ of a portfolio return distribution is the lower ε-quantile of its distribution with negative sign:

$$VaR_\varepsilon(x) = -F_R^{-1}(\varepsilon, x) \tag{1}$$

where $F_R^{-1}(\varepsilon, x) = \inf \{r : F_R(r) > \varepsilon\}$, and F_R^{-1} is the inverse of the portfolio return cumulative distribution function. If R has a multivariate normal distribution with zero means and covariance matrix Σ, then

$$VaR_\varepsilon(x) = \phi^{-1}(\varepsilon)\sigma(x)$$

where $\phi^{-1}(\varepsilon)$ is the $\varepsilon-$quantile of the standard normal distribution, and $\sigma(x) = x^T \Sigma x$.

Below, we briefly describe the RiskMetrics and Historical Filtered Bootstrap strategies (see Sections 2.1 and 2.2 respectively), that are considered as benchmarks to estimate the one-day-ahead VaR. In Section 2.3 we present our model, called Shrinked Volatility VaR

(ShVolVaR), that, as we shall see, include implicitly other two VaR models. Furthermore, in Section 2.4 we briefly report Unconditional Coverage (Kupiec, 1995), Indipendence (Christoffersen, 1998) and Conditional Coverage (Christoffersen and Pelletier, 2004) tests, used to verify advisable features that should be satisfied by a risk model: statistical significance when comparing the observed frequency of violations to the expected one, the independence of violations, and both. Finally, in Section 2.5 we describe the Regulator rules to be validated for the acceptance of a VaR model.

2.1 RiskMetrics VaR model

The assumptions of the RiskMetrics VaR (RiMeVaR) model are that the returns of a generic asset k follow a random walk with independent and identically distributed (i.i.d.) normally distributed changes. More precisely,

$$r_{t,k} = \mu_k + \sigma_{t,k}\eta_{t,k}$$

where $\mu_k = 0$ and $\eta_{t,k} \sim N(0,1)$ is an i.i.d. random perturbation. The returns variance $\sigma_{t,k}$ varies with time and can be estimated by the past information. The RiMeVaR model uses the Exponentially Weighted Moving Average (EWMA) approach to predict volatilities and correlations of the portfolio return. More specifically, volatility forecast of asset k at time $t+1$, given information available at time t, is

$$\sigma_{t+1|t,k} = \sqrt{\lambda\sigma^2_{t|t-1,k} + (1-\lambda)r^2_{t,k}} \tag{2}$$

where $\lambda = 0.94$ for daily data and $\lambda = 0.97$ for monthly data. From Expression (2) it is straightforward to recognize the same formulation of the IGARCH(1,1) model. Furthermore, we have that the one-day-ahead correlation between assets k and j is:

$$\rho_{t+1|t;k,j} = \frac{\sigma_{t+1|t;k,j}}{\sigma_{t+1|t,k}\sigma_{t+1|t,j}}$$

where $\sigma_{t+1|t;k,j}$ is the one-day-ahead covariance forecast between between assets k and j such that $\sigma_{t+1|t;k,j} = \lambda\sigma_{t|t-1;k,j} + (1-\lambda)r_{t,k}r_{t,j}$. Thus, we can define the EWMA covariance matrix as

$$\Sigma_{t+1|t}^{EWMA} = \text{diag}(\sigma_{t+1|t})C_{t+1|t}^{EWMA}\text{diag}(\sigma_{t+1|t})$$

where $\text{diag}(\sigma_{t+1|t})$ is the diagonal matrix with EWMA volatilities of the assets on the diagonal, and $C_{t+1|t}^{EWMA} = \left\{\rho_{t+1|t;k,j}\right\}_{k,j=1,\ldots,n}$ is the EWMA correlation matrix. Therefore, portfolio volatility can be written as

$$\sigma_{t+1|t}(x) = \sqrt{x^T\Sigma_{t+1|t}^{EWMA}x}$$

and the one-day-ahead VaR at confidence level $1-\varepsilon$ as

$$VaR_{t+1|t}(\varepsilon, x) = \phi^{-1}(\varepsilon)\sigma_{t+1|t}(x)$$

where $\phi^{-1}(\varepsilon)$ is the $\varepsilon-$quantile of the standard normal distribution.

2.2 Historical Filtered Bootstrap VaR model

The Historical Filtered Bootstrap (HFB) approach (Barone-Adesi et al, 1999; Brandolini et al, 2001; Zenti and Pallotta, 2000; Marsala et al, 2004) is a mixed procedure in which one represents the market returns using, for instance, an autoregressive moving average generalized autoregressive conditional heteroscedasticity (ARMA-GARCH) model to filter the time series, and then computes the empirical standardized residuals from data without assuming on them any specific probability distribution. Below we give a step-by-step description of HFB procedure.

1. We filter the time series of each asset by an univariate ARMA-GARCH model. More precisely, for the observed returns of the asset k we find the best estimators $\hat{\theta}$ of the

following AR(1)-StudT-GARCH(1,1) model:

$$\begin{aligned} \text{AR(1)} &: \ r_{t,k} = a_k + b_k r_{t-1,k} + \eta_{t,k} \\ \text{StudT-GARCH(1,1)} &: \ \sigma^2_{t,k} = \alpha_k + \beta_k \sigma^2_{t-1,k} + \gamma_k \eta^2_{t-1,k} \\ & \quad\ \eta_{t,k} = \sigma_{t,k} z_{t,k} \end{aligned}$$

where $z_{t,k} = \sqrt{\frac{\nu_k - 2}{\nu_k}} T_{\nu_k}$, T_{ν_k} follows a Student-T distribution with ν_k degrees of freedom, and $\hat{\theta} = \{a_k, b_k, \alpha_k, \beta_k, \gamma_k, \nu_k\}$ are Maximum Likelihood estimators (see, e.g., Vošvrda and Žikeš, 2004, and references therein) obtained on 500 daily data.

2. Using the set of estimators $\hat{\theta} = \{a_k, b_k, \alpha_k, \beta_k, \gamma_k, \nu_k\}$ for all n assets available in the market, we compute from data the standardize residuals $\hat{z}_{t,k}$ with $t = 1, \ldots, T$ and $k = 1, \ldots, n$, i.e., we divide the empirical residuals $\hat{\eta}_{t,k}$ by their estimated volatilities $\hat{\sigma}_{t,k}$.

3. We bootstrap in a parallel fashion the matrix of the empirical standardized residuals $\hat{Z} = \{\hat{z}_{t,k}\}$ with $t = 1, \ldots, T$ and $k = 1, \ldots, n$. More precisely, we randomly sample with replacement the rows of the matrix $\hat{Z}$, thus allowing to capture the multivariate shocks of the entire system.

4. The bootstrapped standardized residuals $\hat{Z}^{boot} = \left\{\hat{z}^{boot}_{s,k}\right\}$, with $s = 1, \ldots, S$ and $k = 1, \ldots, n$, are then used as multivariate innovations in the (univariate) AR(1)-StudT-GARCH(1,1) models to simulate the one-day-ahead returns. In our empirical analysis we employ $S = 10000$ bootstrapped scenarios.

5. Finally, the S scenarios are used to estimate the one-day-ahead VaR at confidence level $1 - \varepsilon$, $VaR_{t+1|t}(\varepsilon, x)$, as in (1).

Note that although AR(1)-StudT-GARCH(1,1) estimations are performed on univariate cases, the dependence structure among the assets is captured by the parallel bootstrap procedure on the standardized residuals $\hat{Z}$. In other words, through this approach of sampling we are able to generate scenarios with historical common shocks. However, for more details see Barone-Adesi et al (1999); Brandolini et al (2001); Zenti and Pallotta (2000); Marsala et al (2004).

2.3 Shrinked Volatility VaR model

We propose here a simple model to forecast ex-ante VaR, assuming, as for RiMeVaR model, that the asset returns are normally distributed with zero mean, but that volatility forecast at time $t+1$, given information available at time t, is the shrinkage between realized and implied volatility. The realized volatility $\hat{\sigma}_{t,k}$ is computed as the standard deviation of the index k returns on 20 stock market days (around 30 calendar days), while the implied volatility $\sigma_{t,k}^{impl}$ is obtained from a basket of call and put options with maturity of 30 calendar days in the market index k. More in detail, we compute the daily implied volatility as $\sigma_{t,k}^{imp} = (256)^{-\frac{1}{2}} V_{t,k}^{imp}/100$, where $V_{t,k}^{imp}$ represents the quoted implied volatility (expressed as a percentage) of the market index k. Thus, in a univariate context we have $r_{t+1|t,k} \sim N(0, \tilde{\sigma}^2_{t+1|t,k}(\alpha))$ with the shrinked volatility

$$\tilde{\sigma}_{t+1|t,k}(\alpha) = (1-\alpha)\hat{\sigma}_{t,k} + \alpha\sigma_{t,k}^{impl} \tag{3}$$

where $\alpha \in (0,1)$ and is called *shrinkage parameter*. On the other hand, in a multivariate context we assume $(r_{t+1|t,1}, r_{t+1|t,2}, \ldots r_{t+1|t,n}) \sim N(0, \tilde{\Sigma}_{t+1|t}(\alpha))$ with the covariance matrix

$$\tilde{\Sigma}_{t+1|t}(\alpha) = \mathrm{diag}(\tilde{\sigma}_{t+1|t}(\alpha))\hat{C}_{t+1|t}\mathrm{diag}(\tilde{\sigma}_{t+1|t}(\alpha))$$

where $\mathrm{diag}(\tilde{\sigma}_{t+1|t}(\alpha))$ is the diagonal matrix with shrinked volatilities of the assets on the diagonal, and $\hat{C}_{t+1|t}$ is the sample correlation matrix estimated on 20 days preceding t. Clearly, the portfolio variance can be written as $\tilde{\sigma}_{t+1|t}(\alpha, x) = \sqrt{x^T\tilde{\Sigma}_{t+1|t}(\alpha)x}$. We then compute the one-day-ahead VaR_ε at $1-\varepsilon$ confidence level for our model, named Shrinked Volatility VaR (ShVolVaR), as follows

$$VaR_{t+1|t}(\varepsilon, \alpha, x) = \phi^{-1}(\varepsilon)\tilde{\sigma}_{t+1|t}(\alpha, x) \tag{4}$$

In our empirical analysis we consider the shrinkage parameter α for different equally-spaced values belonging to the interval $(0, 1)$. Note that if $\alpha = 0$, then Model (4) coincides with the Realized Volatility VaR (ReVolVaR) model; while if $\alpha = 1$ we have the Implied Volatility VaR (ImVolVaR) model.

In Section 3 we will test and compare the ReVolVaR, ImVolVaR and ShVolVaR models with the Historical Filtered Bootstrap VaR (HFBVaR) and the RiskMetrics VaR (RiMeVaR) models that are considered as benchmarks.

2.4 Accuracy Tests

In this section we briefly describe the common tests proposed in the literature to evaluate the statistical accuracy of VaR estimates: the unconditional coverage test (Kupiec, 1995) that analyzes the statistical significance of the observed frequency of violations w.r.t. the expected one; the independence test (Christoffersen, 1998) that gauges the independence of violations, namely the absence of violation clustering; and the conditional coverage test that combines these two desirable properties.

Let us denote by $R_t(x)$ the daily *ex post* portfolio returns with $t = 1, \ldots, T$, and by $VaR_t(\varepsilon)$ the corresponding *ex ante* Value-at-Risk forecasts, where ε is the expected coverage, namely $\mathbf{Pr}_{t-1}(-R_t(x) > VaR_t(\varepsilon)) = \varepsilon$. Let $I_t = \mathbf{1}_{(VaR_t(\varepsilon),+\infty)}(-R_t(x))$ define the random variable hit sequence of $VaR_t(\varepsilon)$ violations, where $\mathbf{1}$ is the indicator function. Note that the hit variable represents only the $VaR_t(\varepsilon)$ violations, excluding any information on their size. Assuming that $I_t \sim Bernoulli(\varepsilon)$ is i.i.d., the Unconditional Coverage (UC) test examines the null hypothesis $H_{0,UC}$ that $\varepsilon = \hat{\varepsilon}$, namely that the observed frequency of violations $\hat{\varepsilon}$ is statistical significant w.r.t. the expected coverage ε. The likelihood function of an i.i.d. hit sequence $I_t \sim Bernoulli(\varepsilon)$ with $t = 1, \ldots, T$ and with a known probability ε that 1 occurs, can be written as:

$$L(I, \varepsilon) = \varepsilon^{N_I}(1 - \varepsilon)^{T - N_I}$$

where $N_I = \sum_{t=1}^{T} I_t$ is the number of $VaR_t(\varepsilon)$ violations. In the case of an i.i.d. Bernoulli variable with unknown probability ε that 1 occurs, it can be estimated by means of the maximum likelihood method as $\hat{\varepsilon} = \frac{N_I}{T}$. Thus, we can obtain the likelihood ratio test of unconditional coverage as

$$LR_{UC} = 2[\ln L(I, \hat{\varepsilon}) - \ln L(I, \varepsilon)]$$

where asymptotically $LR_{UC} \sim \chi^2(\nu = 1)$.

As mentioned above, the UC test assumes that I_t with $t = 1, \ldots, T$ are independent, but this property should be explicitly tested. For this purpose, Christoffersen (1998) provides a test for independence, in which the hit sequence $\{I_t\}_{t=1,\ldots,T}$ follows a first-order Markov chain with switching probability matrix

$$\mathbf{\Pi} = \begin{bmatrix} 1 - \pi_{01} & \pi_{01} \\ 1 - \pi_{11} & \pi_{11} \end{bmatrix}$$

where $\pi_{lq} = \mathbf{Pr}(I_t = q | I_{t-1} = l)$, i.e., the probability that the event l in $t-1$ is followed by the event q in t. The Independence (IND) test examines the null hypothesis $H_{0,IND}$: $\pi_{01} = \pi_{11}$, therefore it investigates on possible violation clustering, namely on eventual repeated deep losses that could cause a bankruptcy. The likelihood function under the hypothesis of the first-order Markov dependence is:

$$L(I; \pi_{01}, \pi_{11}) = (1 - \pi_{01})^{T_{00}} \pi_{01}^{T_{01}} (1 - \pi_{11})^{T_{01}} \pi_{11}^{T_{11}}$$

where T_{lq} represents the number of times that the state l follows the state q. In the case of unknown probabilities π_{01} and π_{11}, they can estimated as $\hat{\pi}_{01} = \frac{T_{01}}{T_{00}+T_{01}}$ and $\hat{\pi}_{11} = \frac{T_{11}}{T_{10}+T_{11}}$. Therefore, the likelihood ratio for the IND test, under the null hypothesis that $\hat{\pi}_{01} = \hat{\pi}_{11} = \hat{\varepsilon}$, can be written as

$$LR_{IND} = 2[\ln L(I, \hat{\pi}_{01}, \hat{\pi}_{11}) - \ln L(I, \hat{\varepsilon})]$$

where $\hat{\varepsilon} = \frac{T_{01}+T_{11}}{T} = \frac{N_I}{T}$, and asymptotically $LR_{IND} \sim \chi^2(\nu = 1)$.

As shown in Christoffersen (1998), these two tests can be combined, determining the

♯ of violations	Action
$[0,4]$	the $VaR_{1\%}$ model is accepted; no actions must be done
$[5,7]$	possible crash of the $VaR_{1\%}$ model; the causes of the violations must be justified and explained
$[8,+\infty)$	the $VaR_{1\%}$ model is not accepted, and it must be changed

Table 1: The Regulator actions related to prefixed intervals of violations of $VaR_{1\%}$ forecasts, required by the UCITS funds.

so-called Conditional Coverage (CC) test, where the null hypothesis $H_{0,CC} : \hat{\pi}_{01} = \hat{\pi}_{11} = \varepsilon$. Clearly, if one of the null hypotheses $H_{0,UC}$ and $H_{0,IND}$ is rejected, even $H_{0,CC}$ will tend to be rejected. For the CC test under the null hypothesis $H_{0,CC}$, the likelihood ratio is

$$LR_{CC} = 2[\ln L(I, \hat{\pi}_{01}, \hat{\pi}_{11}) - \ln L(I, \varepsilon)]$$

where asymptotically $LR_{CC} \sim \chi^2(\nu = 2)$.

2.5 Regulator Backtesting Procedure

The UCITS (Undertaking for Collective Investments in Transferable Securities) mutual funds, under the ESMA's guidelines (CESR, 2010), have to be related to a VaR model with significance level ε =1%. This $VaR_{1\%}$ model, in turn, have to be validated according to specific rules. Indeed, the one-day-ahead $VaR_{1\%}$ forecasts to be accepted have to determine at most 4 violations over the earlier 250 (stock market) days. If a $VaR_{1\%}$ model presents 5, 6 or 7 overshootings[1], risk managers have to declare the violations to the Vigilance, and by means of a documentation they have to explain and to analyze the causes of the model misspecification. If instead a $VaR_{1\%}$ prediction strategy determines 8 or more violations, it is not accepted by the Regulator. Table 1 summarizes the Regulator actions corresponding to the $VaR_{1\%}$ model behavior. More precisely, a $VaR_{1\%}$ model is not discarded if the violations frequency belongs to the interval $[0.4\%; 2.4\%]$ at 95% confidence level (c.l.), or if the violations frequency belongs to the interval $[0\%; 2.8\%]$ at 99% c.l. (see Table 2). The ESMA guidelines require a backtesting procedure at 99% c.l.. Summarizing, a $VaR_{1\%}$ model is considered a good predictive tool up to 4 overshootings,

[1]An overshooting is here a synonym of a VaR violation, a word often used in the financial industry.

Number of hits	Frequency (%)	p-value (%)
0	0.0	2.5
1	0.4	27.8
2	0.8	74.2
3	1.2	75.8
4	1.6	38.0
5	2.0	16.2
6	2.4	5.9
7	2.8	1.9
8	3.2	0.5

Table 2: Number of violations, corresponding frequency on 250 days, and related p-value for the UC test.

if instead during the last 250 business days the number of overshootings for each UCITS exceeds 4 hits, then the senior management team must be informed. The competent authority may take specific actions, and may apply some limitations for the use of the $VaR_{1\%}$ model, when the overshootings exceed an unacceptable number of violations. Indeed, if the observed hit frequency is higher than 2.80% (7 overshootings), then some kind of action has to be taken in order to reduce the risk model misspecification. On the other hand, if there are no violations, nothing has to be done.

In the empirical analysis, reported in the next section, to have an early warning on model performances, we decided to be more restrictive than the ESMA rules adopting a backtesting procedure at 95% c.l.. Therefore, in this work the maximum number of admissible overshootings is 6. Furthermore, from Table 2 note that the case of absence of violations (0 hits) is not within the region of acceptance for the UC test at 95% c.l..

3 Empirical analysis

In this section we present computational results for five models: the Implied Volatility VaR (ImVolVaR), the Realized Volatility VaR (ReVolVaR), the Shrinked Volatility VaR (ShVolVaR), the Historical Filtered Bootstrap VaR (HFBVaR), and the RiskMetrics VaR (RiMeVaR) models. The analysis is performed on the S&P 500, Eurostoxx50, Dax, FTSE100 and Topix market indexes, in which, in addition to the index values, are also available the corresponding implied volatilities. The one-day-ahead VaR forecasts

Market	Index ticker	Implied Volatility ticker	Start date	End date	daily VaR forecasts
SP500	SPTR	VIX	30/01/1990	30/9/2015	6472
Eurostoxx50	SX5T	V2X	01/02/1999	30/9/2015	4265
DAX	DAX	V1X	30/01/1992	30/9/2015	5994
FTSE100	TUKXG	VFTSE	01/02/2000	30/9/2015	3959
Topix	TPXDDVD	VXJ	02/03/1998	30/9/2015	4320

Table 3: List of the data sets analyzed.

obtained by the five models are validated both by considering each individual index and portfolios of such indexes, namely both in the univariate and multivariate context. As reported in Table 3, the lengths of the indexes time series, consisting of daily values obtained from Bloomberg, cover different time windows according to the availability of the implied volatility values. Each data set has the same end date on September 30, 2015, is expressed in local currency, and follows its own financial calendar. Furthermore, the start dates of each data set, shown in Table 3, refer to the starting points of the VaR forecasts.

More in detail, we adopt a working days calendar, and on these days we take the prices of all market indexes. If a market is closed on a specific day, e.g., for holidays, then for that day we replicate the price with the last available value. Clearly, on that particular day the index return will be zero. This pre-processing is required to compute a fair analysis of correlations among indexes, i.e., to avoid possible lags among the returns time series. Conversely, the realized volatilities are estimated on the original time series without this pre-processing, because, unlike the correlations estimates, those of volatilities would be too influenced by this pre-processing.

For completeness, we also list in Table 3 the tickers corresponding to each time series. In the case of VaR forecasts analysis for portfolios of the five indexes, we consider the maximum time window covered by all data sets, namely from February 1, 2000 to September 30, 2015.

3.1 Computational Results

In this section we discuss the main results on the behavior of the VaR models in the univariate (Section 3.1.1) and in the multivariate (Section 3.1.2) framework.

3.1.1 Univariate framework

In Tables 4 and 5 we provide the p-values (expressed as a percentage) of the Unconditional Coverage (UC), Independence (IND) and Conditional Coverage (CC) tests. We report in dark-gray the cases in which we can not accept the null hypotheses (i.e., the VaR model is not able to capture the expected frequency of violations (UC), or suffer from dependence of violations (IND), or both (CC)). In light-gray we highlight the cases in which the null hypothesis is accepted at 99% confidence level (c.l.), but rejected at 95% c.l.. The rest of the accepted cases are reported in bold.

Table 4 shows that the RiMeVaR model does not suffer from dependence of violations, but fails to capture the UC hypothesis, except for the Topix index. The HFBVaR model is within the region of acceptance of the UC test at 99% c.l. for all indexes, while considering 95% c.l. we observe 2 rejections (out of 5) of the IND hypothesis, and consequently 2 rejections of the CC hypothesis. It is evident that the HFBVaR strategy shows better results than those of RiskMetrics. The ImVolVaR model ($\alpha = 1$) fails the UC test 4 times out of 5, and the CC test 3 times at 99% c.l., and only in 3 cases it does not suffer from dependence of violations. The ReVolVaR model ($\alpha = 0$) presents the worst results, indeed for all indexes the null hypothesis of the UC and CC tests is not accepted.

Index	ImVolVaR ($\alpha = 1$)			ReVolVaR ($\alpha = 0$)			HFBVaR			RiMeVaR		
	UC	IND	CC	UC	IND	CC	UC	IND	CC	UC	IND	CC
SP500	0.00	**56.08**	0.01	0.00	2.03	0.00	**5.03**	2.21	1.07	0.00	**25.17**	0.00
EuroStoxx50	0.57	**57.22**	1.88	0.00	**86.55**	0.00	**16.42**	**66.74**	**34.65**	0.00	**53.75**	0.00
DAX	0.37	0.18	0.01	0.00	**44.59**	0.00	**7.83**	**31.88**	**12.91**	0.00	**56.65**	0.00
Ftse100	3.29	**17.94**	4.17	0.00	0.33	0.00	**14.73**	**63.93**	**31.35**	0.00	**18.79**	0.00
Topix	0.25	0.78	0.03	0.00	0.12	0.00	**6.12**	4.15	2.17	**62.03**	**5.70**	**14.45**

Table 4: The p-values (%) of the UC, IND, and CC tests for the ImVolVaR, ReVolVaR, HFBVaR, and RiMeVaR models.

In Table 5 we present a selection of results for the ShVolVaR model with different representative values of the *shrinkage parameter*, namely $\alpha = \{1/3, 1/2, 2/3\}$. Clearly, low values of α tend to give more importance to the realized volatility, while for high α the one-day-ahead VaR estimation mainly depends on the implied volatility. Considering the ShVolVaR model with $\alpha = 1/3$, we observe 2 rejections for the UC test at 99%

confidence level (c.l.), and 1 rejection for the CC test. For $\alpha = 2/3$, the ShVolVaR model presents 1 rejection for the UC and CC tests at 99% c.l., and only in 2 cases out of 5 the null hypothesis of the IND and CC tests at 95% c.l. is accepted. The best results are obtained by the ShVolVaR model with $\alpha = 1/2$, where therefore the shrinked volatility $\tilde{\sigma}_{k,t}$ in (3) can be simply interpreted as the average between the realized and the implied volatility of the asset k. In Tables 6 and 7 we report the frequency related to the observed

Index	ShVolVar ($\alpha = 1/3$)			ShVolVar ($\alpha = 1/2$)			ShVolVar ($\alpha = 2/3$)		
	UC	IND	CC	UC	IND	CC	UC	IND	CC
SP500	**5.0**	**38.0**	**10.0**	**55.1**	**12.9**	**26.5**	0.6	3.7	0.3
EuroStoxx50	**12.5**	**69.2**	**28.5**	**95.7**	**45.7**	**75.7**	**68.0**	**39.3**	**63.7**
DAX	0.5	3.4	0.2	**10.1**	**7.4**	**5.3**	**60.5**	1.6	4.9
Ftse100	0.6	**27.9**	1.3	**31.8**	**56.5**	**51.5**	**67.6**	**36.0**	**60.2**
Topix	**56.7**	**10.9**	**23.4**	**62.0**	**5.7**	**14.5**	**10.4**	2.6	2.2

Table 5: The p-values (%) of the Unconditional Coverage, the Independence, and the Conditional Coverage tests for the ShVolVaR models with $\alpha = \{1/3, 1/2, 2/3\}$.

number of VaR violations on the three overshooting intervals required by the ESMA rules (see Section 2.5). We can observe that the RiMeVaR model shows a high percentage of times where the overshootings are more than 6 with the only exception of the Topix index. The other benchmark model (HFBVaR) has a frequency of violations on the interval $[7, +\infty)$ that is not greater than 5.1% for the different indexes. The ImVolVaR model, overestimating VaR, presents good performance according the Regulator rules, indeed we observe that the frequency of violations on the interval $[7, +\infty)$ is at most 2%. Conversely, for the same overshooting interval the ReVolVaR model has poor performance, showing the frequency of violations always greater than 26.1%. The new proposed model (ShVolVaR with $\alpha = 0.5$) presents a frequency of overshootings that is not greater than 3% on the interval $[7, +\infty)$. Therefore, from the Regulator viewpoint the ShVolVaR model does not require any modification.

Regarding the case of 0 violations, it is allowed by the Regulator, but from the investor viewpoint this means the mismanagement of capital. Furthermore, as shown in Table 2 the case of 0 hits is always rejected from the UC test with 95% confidence level. However, the VaR models that best minimize the frequency of absence of violations on periods of 250 days are ReVolVaR and RiMeVaR that are aggressive strategies to estimate the one-day-ahead VaR, while the worst model is ImVolVaR that is a strategy too conservative. For 0 hits the performance of the ShVolVaR model is comparable to that of HFBVaR, indeed ShVolVaR shows that the average of frequency violations on the five indexes is equal to 5.6%, while for HFBVaR the average is 4.2%. The only two models analyzed that comply the Regulator requests and that satisfy the accuracy tests, are HFBVaR and ShVolVaR. We point out, therefore, that our naive model substantially reaches good

VaR	SP500				Topix			
Model	$[0, 4]$	$[5, 6]$	$[7, +\infty)$	0 hits	$[0, 4]$	$[5, 6]$	$[7, +\infty)$	0 hits
ImVolVaR	96.1	1.8	2.0	34.4	99.2	0.8	0.0	29.5
ReVolVaR	38.9	23.5	37.5	0.4	36.2	35.7	28.1	0.0
ShVolVaR	89.8	7.2	3.0	14.4	94.9	5.1	0.0	5.4
HFBVaR	83.9	12.8	3.3	6.1	77.6	19.9	2.5	4.4
RiMeVaR	39.8	28.9	31.3	0.4	94.9	5.1	0.0	5.4

Table 6: Observed frequencies of VaR violations on the three representative overshooting intervals required by the ESMA rules for the SPX and TOPIX indexes.

VaR	EuroStoxx50				DAX				Ftse100			
Model	$[0, 4]$	$[5, 6]$	$[7, +\infty)$	0 hits	$[0, 4]$	$[5, 6]$	$[7, +\infty)$	0 hits	$[0, 4]$	$[5, 6]$	$[7, +\infty)$	0 hits
ImVolVaR	96.9	3.1	0.0	19.0	98.9	1.1	0.0	25.1	95.4	4.6	0.0	30.7
ReVolVaR	36.7	37.2	26.1	0.0	32.9	39.3	27.8	1.3	21.8	33.6	44.6	0.0
ShVolVaR	91.5	8.4	0.0	3.2	87.2	12.3	0.5	3.7	88.5	10.8	0.8	1.3
HFBVaR	84.6	15.4	0.0	1.0	83.1	16.8	0.0	3.1	85.0	9.9	5.1	6.6
RiMeVaR	40.5	33.4	26.2	0.0	43.8	42.2	14.0	1.3	34.0	34.3	31.7	0.0

Table 7: Observed frequencies of VaR violations on the three representative overshooting intervals required by the ESMA rules for the SX5E, DAX and FTSE100 indexes.

performance results such as the HFBVaR model that seems to be one of the best methods to estimate VaR (Abad et al, 2014).

3.1.2 Multivariate framework

In this section we report the empirical results for multivariate VaR estimations on 20 portfolios, listed in Table 8, and commonly considered as benchmark portfolios in the asset management industry (see, e.g., MSCI, 2015; Towers-Watson, 2015). The portfolios are composed by a number of indexes ranging from 2 to 5. Furthermore, we present here

Portfolio	Weights (%)				
name	SP500	EuroStoxx50	DAX	Ftse100	Topix
BMK1	50.0	0.0	0.0	25.0	25.0
BMK2	50.0	50.0	0.0	0.0	0.0
BMK3	66.0	34.0	0.0	0.0	0.0
BMK4	0.0	50.0	0.0	0.0	50.0
BMK5	10.0	10.0	10.0	10.0	60.0
BMK6	34.0	33.0	0.0	0.0	33.0
BMK7	0.0	33.0	0.0	33.0	34.0
BMK8	50.0	25.0	0.0	25.0	0.0
BMK9	0.0	35.0	25.0	40.0	0.0
BMK10	0.0	50.0	0.0	50.0	0.0
BMK11	10.0	60.0	10.0	10.0	10.0
BMK12	50.0	0.0	50.0	0.0	0.0
BMK13	25.0	25.0	0.0	25.0	25.0
BMK14	50.0	25.0	0.0	0.0	25.0
BMK15	0.0	0.0	50.0	50.0	0.0
BMK16	12.5	30.0	5.0	30.0	12.5
BMK17	50.0	20.0	5.0	15.0	10.0
BMK18	10.0	25.0	20.0	35.0	10.0
BMK19	20.0	20.0	20.0	20.0	20.0
BMK20	50.0	0.0	0.0	0.0	50.0

Table 8: List of the 20 benchmark portfolios weights used for the multivariate VaR estimations analysis.

only the empirical analysis for those models that in the univariate framework have shown the best results, namely HFBVaR and ShVolVaR with $\alpha = 0.5$.

In Table 9 we report the p-values obtained by the HFBVaR and ShVolVaR models for the UC test, that is the accuracy test on which the Regulator is mainly interested. Following the convention described in Section 3.1.1, we note that using the ShVolVaR model 19 portfolios out of 20 pass the UC test at 95% c.l. (in bold), while for HFBVaR only 13 portfolios out of 20. If we consider the UC test at 99% c.l. (in light-gray), then for all the benchmark portfolios the VaR estimations obtained by the ShVolVaR and the HFBVaR models show statistical significance when comparing the observed frequency of VaR violations w.r.t. the expected one. In Table 10 for each benchmark portfolio we report the frequency (%) related to the number of VaR violations, achieved by the

Portfolio name	UC test → p-value (%)	
	ShVolVaR	HFBVaR
BMK1	**98.2**	2.4
BMK2	**89.2**	**42.8**
BMK3	**85.8**	**21.5**
BMK4	**73.9**	**34.6**
BMK5	**73.9**	**27.5**
BMK6	**73.9**	3.5
BMK7	**62.6**	**12.5**
BMK8	**62.6**	**42.8**
BMK9	**42.8**	**34.6**
BMK10	**34.6**	**9.3**
BMK11	**34.6**	3.5
BMK12	**27.5**	**27.5**
BMK13	**27.5**	**16.5**
BMK14	**21.5**	3.5
BMK15	**16.5**	**34.6**
BMK16	**16.5**	4.9
BMK17	**16.5**	**34.6**
BMK18	**12.5**	2.4
BMK19	**12.5**	3.5
BMK20	3.5	**16.5**

Table 9: The p-values (%) of the UC test obtained by the HFBVaR and the ShVolVaR models for the 20 benchmark portfolios.

HFBVaR and the ShVolVaR models, on the three representative overshooting intervals. For each interval and for each portfolio, the best result is marked in bold. In the case of

Portfolio name	[0, 4]		[5, 6]		[7, +∞)	
	ShVolVaR	HFBVaR	ShVolVaR	HFBVaR	ShVolVaR	HFBVaR
BMK1	**89.5**	72.4	**10.1**	22.3	**0.3**	5.4
BMK2	88.8	**89.9**	**9.1**	9.7	2.1	**0.3**
BMK3	**94.1**	85.8	**5.5**	13.1	**0.3**	1.1
BMK4	**92.6**	91.2	**7.0**	8.4	0.4	0.4
BMK5	**94.0**	90.0	**6.0**	10.0	0.0	0.0
BMK6	**86.1**	78.9	**13.9**	18.1	**0.0**	3.0
BMK7	**88.6**	83.6	**9.8**	14.7	**1.6**	1.7
BMK8	86.0	**90.4**	**8.6**	9.3	5.4	**0.3**
BMK9	**93.8**	93.5	**5.5**	6.5	0.7	**0.0**
BMK10	**87.5**	82.3	**10.2**	17.5	2.3	**0.2**
BMK11	**82.1**	78.5	17.2	**16.9**	**0.7**	4.7
BMK12	**87.9**	86.5	**9.9**	13.2	2.3	**0.3**
BMK13	80.3	**84.3**	18.2	**15.0**	1.4	**0.8**
BMK14	79.4	**82.6**	20.0	**9.0**	**0.7**	8.4
BMK15	83.7	**88.7**	14.0	**11.1**	2.3	**0.2**
BMK16	80.7	**83.8**	16.8	**15.2**	2.5	**1.0**
BMK18	84.5	**88.0**	13.0	**8.9**	**2.5**	3.1
BMK19	**81.3**	79.2	**17.3**	18.5	**1.4**	2.3
BMK20	80.6	**86.3**	17.9	**12.8**	1.5	**0.9**
Mean	**86.2**	84.8	**12.2**	13.4	**1.6**	1.8
Min	**79.4**	72.4	**5.5**	6.5	0.0	0.0
Max	**94.1**	93.5	**20.0**	22.3	**5.4**	8.4

Table 10: Observed frequencies of VaR violations on the three representative overshooting intervals required by the ESMA rules for the 20 portfolios listed in Table 8.

VaR violations greater than 6, the ShVolVaR model performs better than HFBVaR in 8 cases out of 20, on 2 benchmark portfolios the frequencies are equal, while in 10 cases out of 20 HFBVaR presents better results than ShVolVaR. Therefore, there is no apparent relation of dominance among the two approaches. The average of the frequencies on the 20 portfolios is below 2% for both models, namely in very few cases the ShVolVaR and the HFBVaR models require adjustments.

In the overshooting interval $[0, 4]$, the average frequency is 86.2% for ShVolVaR, and

84.8% for HFBVaR. For the intermediate interval $[5, 6]$ the ShVolVaR and the HFBVaR models show average frequencies of 12.2% and of 13.4%, respectively. Thus even in these cases the two approaches seems to have similar performance.

We stress here that the ideal model, both from the Regulator and from the investor viewpoint, should have 100% of the VaR violations on the interval $[1, 4]$. Table 11 shows

Portfolio	0 hits		$[1, 4]$	
name	ShVolVaR	HFBVaR	ShVolVaR	HFBVaR
BMK1	8.8	2.4	80.8	70.0
BMK2	8.4	3.6	80.4	86.3
BMK3	5.7	3.6	88.5	82.2
BMK4	4.9	6.9	87.6	84.3
BMK5	1.5	1.5	92.4	88.5
BMK6	7.9	1.7	78.2	77.1
BMK7	4.2	2.7	84.4	80.9
BMK8	1.9	3.6	84.0	86.7
BMK9	1.2	0.8	92.6	92.7
BMK10	1.2	1.2	86.3	81.1
BMK11	7.4	0.8	74.7	77.7
BMK12	5.7	3.6	82.2	82.9
BMK13	2.0	2.1	78.4	82.1
BMK14	7.9	2.2	71.5	80.4
BMK15	0.6	2.0	83.1	86.6
BMK16	0.8	0.8	80.0	83.0
BMK18	2.6	0.0	79.5	81.3
BMK19	0.8	0.0	80.5	79.2
BMK20	1.0	5.5	79.5	80.8
Mean	3.7	2.5	82.4	82.4
Min	0.2	0.0	71.5	70.0
Max	8.8	6.9	92.6	92.7

Table 11: Observed frequencies of the number of VaR violations on the interval $[1, 4]$, and in the case of absence of violations (0 hits) for the 20 benchmark portfolios.

that on the interval $[1, 4]$ the average frequencies of ShVolVaR and of HFBVaR are both equal to 82.4%, and that also the minimum and maximum frequencies are similar, around 70.0% and 92.7%.

We can conclude that the forecasting power of our naive model is definitely comparable with that of one of the best methods to estimate VaR. However, while the VaR forecasts of HFBVaR are based on a fairly sophisticated procedures, those of the ShVolVaR model are extremely simple to obtain.

To better highlight the forecasting power in the multivariate framework, we also test the ShVolVaR model on 2000 portfolios uniformly distributed on the unit simplex, i.e., portfolios with weights that must sum to 1 without short-sellings. To generate these 2000 portfolios, we employ an algorithm provided by Rubinstein (1982). In Table 12 we report some statistics (mean, median, min and max) of the observed frequencies (also called the empirical coverages) of the VaR violations and of the p-values obtained by the UC test for the ShVolVaR model. The empirical coverages range from 0.91% to 1.35% with the median equal to 1.17%, while the p-values are between 3.37% and 98.28% with the median equal to 27.47%. This means that the UC test at 99% c.l. never rejects the null hypothesis, while at 95% c.l. the UC hypothesis is rejected in 17 cases out of 2000. Table 12 also provides some statistics (mean, median, min and max) of the observed frequencies related to the number of VaR violations on the usual three overshooting intervals required by the UCITS rules. The results obtained by the ShVolVaR model for 2000 randomly generated portfolios show a behavior similar to that achieved from the restricted set of the 20 benchmark portfolios. So even this last analysis confirms that our simple prediction strategy to model VaR has very promising performance. Hence, due to its ease of implementation, the ShVolVaR model could be used as a tool for portfolio managers to quickly monitor investment decisions before employing more sophisticated risk management systems.

	Empirical	UC test	Overshooting intervals			
	coverage	p-value	$[0,4]$	$[5,6]$	$[7,+\infty)$	0 hits
Mean	1.17	33.32	86.2	12.3	1.5	2.7
Min	0.91	3.47	72.8	1.1	0.0	0.0
Max	1.35	98.24	98.9	25.5	7.3	13.2
Median	1.17	27.47	86.1	12.3	1.3	2.0

Table 12: Mean, median, min and max of the realized frequency of the VaR violations and the p-value obtained by the UC test for the ShVolVaR model on 2000 portfolios uniformly distributed on the unit simplex.

4 Conclusions

In this work we proposed a new method to predict VaR, both using variables known on the market (implied volatilities) and variable estimated on data (realized volatilities). The main idea behind our approach is to use a combination of information both on the expected future risk and on the past estimated risk. The forecasting power of our ShVolVaR model is compared with that of several models proposed in the literature, including one of the best methods to estimate VaR, such as the HFBVaR model. All models are tested both on the statistical accuracy (by means of the Unconditional Coverage, the Independence and the Conditional Coverage tests) and on efficiency (by means of the backtesting procedure of the Vigilance). Furthermore, they are validated both on individual asset and on portfolios of such assets, namely both in the univariate and multivariate framework.

Although the ShVolVaR model is based on strong assumptions such as those of RiskMetrics, namely the one-day-head returns are normally distributed with zero mean, its forecasting power is comparable to that of the more sophisticated HFBVaR model. Thus, we provide a fast and simple tool that can be also implemented on a common spreadsheet, which, for instance, could be directly integrated with data providers.

Talking in practical terms, in this paper we examine the case of a portfolio manager who administrates a flexible UCITS fund, aiming to obtain the maximum return with a constraint on risk, measured by VaR. Since the portfolio manager must support transition costs when he buys or sells assets, before performing the trading he could use our quick tool of forecasting as what-if scenario analysis. If the portfolio VaR is within specific risk bounds, the portfolio manager could purchase and sale; otherwise he should revise his investment. Therefore, this pre-analysis obtained by our model can allow the control of risk both upstream and downstream of the investment process. Indeed, the asset manager typically constructs his portfolio, and only afterwards the risk manager ensures compliance with the risk limits. So if the portfolio VaR goes out of the Regulator's limitations, then the portfolio manager has to change its investment strategy, thus leading to support twice the trading costs.

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